DTU Library

Calculating the sensitivity of wind turbine loads to wind inputs using response surfaces

Rinker, Jennifer M.

Published in:
Journal of Physics: Conference Series (Online)

Link to article, DOI:
10.1088/1742-6596/753/3/032057

Publication date:
2016

Document Version
Publisher's PDF, also known as Version of record

Calculating the sensitivity of wind turbine loads to wind inputs using response surfaces

2016 J. Phys.: Conf. Ser. 753 032057

(http://iopscience.iop.org/1742-6596/753/3/032057)

Calculating the sensitivity of wind turbine loads to wind inputs using response surfaces

Jennifer M. Rinker[1]

[1] Technical University of Denmark, Department of Wind Energy, Frederiksborgvej 399, Building 114 Room S01, 4000 Roskilde

E-mail: rink@dtu.dk

Abstract. This paper presents a methodology to calculate wind turbine load sensitivities to turbulence parameters through the use of response surfaces. A response surface is a high-dimensional polynomial surface that can be calibrated to any set of input/output data and then used to generate synthetic data at a low computational cost. Sobol sensitivity indices (SIs) can then be calculated with relative ease using the calibrated response surface. The proposed methodology is demonstrated by calculating the total sensitivity of the maximum blade root bending moment of the WindPACT 5 MW reference model to four turbulence input parameters: a reference mean wind speed, a reference turbulence intensity, the Kaimal length scale, and a novel parameter reflecting the nonstationarity present in the inflow turbulence. The input/output data used to calibrate the response surface were generated for a previous project. The fit of the calibrated response surface is evaluated in terms of error between the model and the training data and in terms of the convergence. The Sobol SIs are calculated using the calibrated response surface, and the convergence is examined. The Sobol SIs reveal that, of the four turbulence parameters examined in this paper, the variance caused by the Kaimal length scale and nonstationarity parameter are negligible. Thus, the findings in this paper represent the first systematic evidence that stochastic wind turbine load response statistics can be modeled purely by mean wind wind speed and turbulence intensity.

1. Introduction

As the state of the art of wind energy has progressed, the size of the average commercial wind turbine has increased drastically. This increase in wind turbine size is primarily driven by the increased power production that large rotors yield. The increase in wind turbine size has increased complications in terms of transportation, construction, and design. Additionally, the cost of larger components has increased, which means that the consequences of failure are more severe. Thus, as the wind turbine size has increased, the importance of adequately designing the system has also increased. To ensure proper wind turbine design, the turbulence and turbine must both be characterized with a sufficient degree of accuracy, and the connection between the two dynamical systems must be understood.

Despite the importance of characterizing turbulence-turbine interactions, there are only a select number of papers in the literature that examine the effects of atmospheric parameters on the loads and generated power in a wind turbine. Of the existing literature, some focus on atmospheric stability or related phenomena (e.g., [1, 2, 3, 4]), some focus on specific atmospheric parameters (e.g., [5, 6, 7], and still others focus on statistical properties (e.g., [8]). The approaches in the referenced literature are varied, but there is lacking a systematic analysis to determine which atmospheric parameters have the most significant effects on wind turbine loads and which can be set to nominal values.

This paper seeks to fill this gap in the state of the art by utilizing response surfaces to calculate the sensitivity of wind turbine load response statistics to a set of common turbulence parameters. The sensitivities can be used to quantify which turbulence parameters have the largest effect on wind turbine loads and which are negligible. The load sensitivity can be quantified through a global sensitivity metric such as the Sobol sensitivity indices (SIs) [9]. A Sobol sensitivity analysis is a variance-based sensitivity analyses that calculates the sensitivity of an output quantity (e.g., a wind turbine load response statistic) to a specific subset of input parameters (e.g., select turbulence parameters). The calculated SIs provide information on which input parameters have the largest effect on the output parameters, and if a calculated SI is sufficiently low, then the corresponding input parameters can be deemed "not important" and set to nominal values [10]. Thus, Sobol SIs can be used not only to quantify the relative importance of input parameters but also for model reduction.

The main drawback of the Sobol SIs is that many model evaluations are required to calculate the integrals in the SI equation [11]. One method to mitigate this issue is to use a surrogate model that can easily generate synthetic model data, such as a reponse surface. Response surfaces are a surrogate modeling technique in which a multi-dimensional polynomial is used to map a set of model inputs $\mathbf{x} = [x_1, x_2, \ldots, x_n]$ to a scalar output y [12]:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 \cdots . \tag{1}$$

Response surfaces are extremely easy to calibrate and can generate synthetic data at very little computational expense, as the model is purely arithmetic once the coefficients have been calibrated. Thus, a properly calibrated response surface can be used to calculate Sobol SIs with ease. While this technique has been used in the literature to calculate SIs in fields outside of wind energy [13, 14], to the author's knowledge it has never been utilized in wind energy applications. The objective of this paper is therefore to demonstrate the ability of response surfaces and Sobol SIs to determine which turbulence parameters are important to consider in load analyses. A single wind turbine model and load response statistic (viz. the maximum blade root bending moment of the WindPACT 5 MW reference model [15]) is examined here for brevity, but the proposed methodology is applicable to any wind turbine model or response statistic.

The remainder of the paper is laid out as follows. First, a description of the training data that was used to calibrate the response surfaces is provided in Sec. 2. The training data were generated as part of a previous project, and the section details the wind turbine and turbulence models that were used to generate the input/output data. Next, Sec. 3 demonstrates the proposed fitting methodology on the 10-minute maximum blade-root bending moment for the WindPACT 5 MW reference model. An evaluation of the model fit and response surface convergence is also presented, along with the procedure to sample stochastic load response statistics from the calibrated response surface. The calibrated response surface is used in Sec. 4 to calculate Sobol sensitivity indices to determine which turbulence parameters are important to model and which can be set to nominal values. Lastly, the conclusions of this paper are presented in Sec. 5.

2. Background

The wind turbine models and training data used in this paper were available from a previous project. This section provides a detailed description of the models and the methodology that were used to generate the training data to which the response surfaces were calibrated.

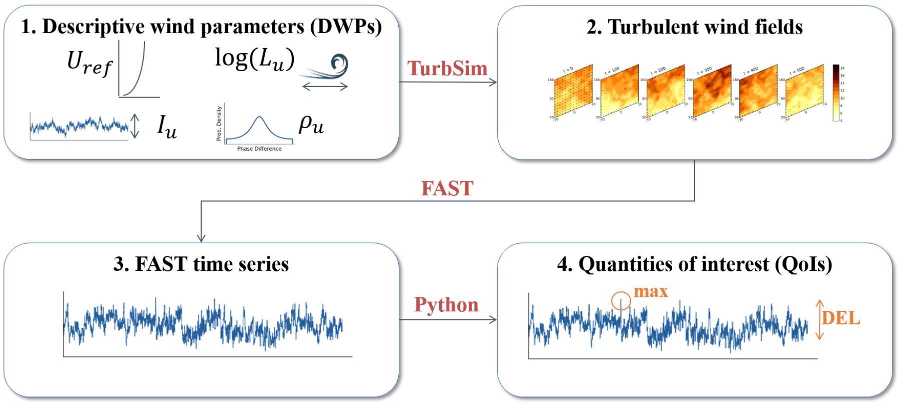

Figure 1. Procedure to generate input/output data to calibrate response surface

2.1. Wind turbine model

The wind turbine model used in this paper is the WindPACT 5 MW reference model. The model was originally developed as part of the WindPACT rotor scaling study in the early 2000s [15], and it is one of a set of four identical models that were scaled to different sizes (750 kW, 1.5 MW, 3 MW, and 5 MW). The model is a three-bladed, upwind configuration with a hub height of 154 m and a blade length of 60 m. The blades were designed in a separate WindPACT project [16] and consist of triaxial fiberglass laminate with a balsa wood core. The airfoils are the National Renewable Energy Laboratory (NREL) S-series airfoils [17] that were scaled and given finite trailing-edge thickness. The controller is a variable-speed, pitch-to-feather configuration with a simple quadratic controller below rated and a gain-scheduled PID controller above rated [15, Appendix E]. Detailed descriptions of the wind turbine model can be found in [15].

2.2. Training data

The training data used to calibrate the response surface in this paper were generated for a previous project. The project used NREL's high-performance computing (HPC) resources to simulate thousands of nonstationary turbulent fields, to use those fields as input to the WindPACT reference models, then to calculate and save selected statistics from the wind turbines' responses. An overview of the procedure used to generate the training data is shown in Fig. 1. The details of the training data generation procedure, including the atmospheric and turbine modeling, are discussed below.

2.2.1. Simulation of nonstationary turbulence The objective of the previous project was to quantify the importance of inflow nonstationarity on wind turbine loads. This nonstationarity was modeled through a novel methodology called temporal coherence, which can produce stochastic turbulent fields with coherent packets of energy. For brevity, only an overview of temporal coherence is provided here; the interested reader is referred to [18] for more detail.

Temporal coherence is a correlation between Fourier phases at different frequencies for the same spatial location, and it is easily implemented in simulation through the use of "phase difference distributions" (PDDs). A PDD is a prescribed probability distribution on the difference in phase between adjacent Fourier frequencies f_i and f_{i+1} for any given location in the spatial grid. Prescribing a PDD that is non-uniform (i.e., that has some degree of concentration) results in a correlation between the Fourier phases at different frequencies for that point in space. The more concentrated the probability distribution—where the concentration is measured by

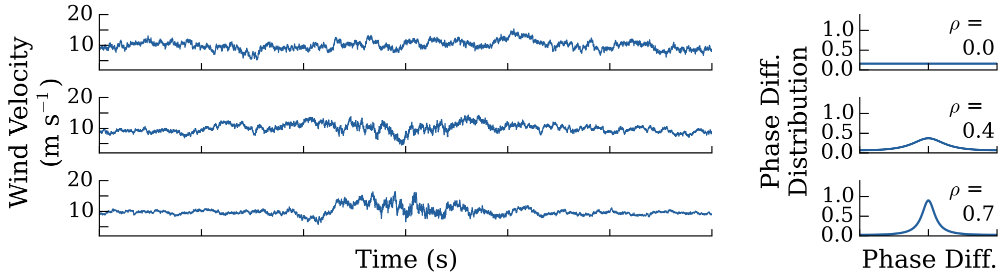

Figure 2. Demonstration of the effect of the phase difference distribution (right column) on the simulated time history (left column). The top, middle, and bottom rows have concentration parameters parameters equal to 0, 0.4, and 0.7, respectively. All spectra generated using a Kaimal spectrum with $U_{ref} = 10$ m s^{-1}, $\sigma_u = 1.4$ m s^{-1}, and $L_u = 340.2$ m.

the "concentration parameter" ρ—the more temporal coherence and the more nonstationary the simulated time history. This effect is demonstrated in Fig. 2, which has PDDs of varying concentration in the right column and the corresponding simulated turbulent time series in the left column. Higher degrees of concentration in the PDD produce more energy concentration in the time domain, though all simulations utilize the same Kaimal power spectrum. Thus, the concentration parameter ρ is a direct controller of the nonstationarity in the atmospheric simulations.

The full-field nonstationary turbulence was generated in the previous project using a customized version of TurbSim v2.0, NREL's open-source stochastic turbulence simulator. The source code was modified to allow the specification of temporal coherence in all three turbulent components (u, v, and w), but the remaining model options were assumed to be the same as the Kaimal Spectrum with Exponential Coherence (KSEC) model from wind turbine design standard IEC 61400-1, Annex B [19]. The KSEC model has eight turbulence parameters: a reference 10-minute mean wind speed (U_{ref}), the turbulence standard deviation for the three turbulence components (σ_u, σ_v, and σ_w), the Kaimal length sale for the three turbulence components (L_u, L_v, and L_w), and a coherence length scale (L_c). However, the design standard recommends the following relationships for hub heights greater than 60 m [19]:

$$\begin{array}{lll} \sigma_v = 0.8\sigma_u & L_v = 0.33L_u & L_c = 340.2\ m \\ \sigma_w = 0.5\sigma_u & L_w = 0.08L_u & \end{array} \tag{2}$$

Thus, there are only three unique turbulence parameters for the KSEC model: U_{ref}, σ_u, and L_u. To add temporal coherence, it can be assumed that $\rho_u = \rho_v = \rho_w$ because previous work has determined that the temporal coherence in the different components is extremely highly correlated [18]; thus, only one extra turbulence parameter, ρ_u, must be prescribed for a model with temporal coherence. Assuming the spatial grid and time step are known *a priori*, the set of turbulence parameters that therefore define a KSEC model with temporal coherence are U_{ref}, σ_u, L_u, and ρ_u. An alternative but equivalent formulation is to define the reference turbulence intensity $I_{u,ref} = \sigma_u/U_{ref}$ in place of the turbulence standard deviation.

The previous project sought to create a dataset that could be used to map these four turbulence parameters (U_{ref}, $I_{u,ref}$, L_u, and ρ_u) to wind turbine response statistics. To this end, the input parameter space was discretized into a four-dimensional hypergrid with the following edge vectors:

$$\begin{array}{rcl} U_{ref} & = & [5, 7, 9, 10, 10.5, 11, 11.5, 12, 13, 16, 19, 22] \\ I_{u,ref} & = & [0.1, 0.2, 0.3, 0.4, 0.5] \\ \log_{10}(L_u) & = & [1.5, 2, 2.5, 3] \\ \rho_u & = & [0.0, 0.1, 0.2, 0.3, 0.4] \end{array} \tag{3}$$

The values for U_{ref} were chosen with respect to a reference height of 90 m and to be within the cut-in/cut-out wind speed ranges for all four WindPACT turbines. Extra points were simulated near the rated wind speed (approximately 11.5 m s^{-1}) to capture interesting phenomena that might occur due to the transition between controller regions. The turbulence intensity values were chosen to encompass the majority of values that were calculated from an atmospheric dataset recorded at meteorological tower M4 at NREL near Boulder, Colorado, USA [20, 21]. The logarithm of L_u was used instead of L_u to better regularize the input data for subsequent analyses, and the values were also chosen to encompass the majority of values calculated from the M4 dataset. Lastly, the values of ρ_u were chosen to include $\rho_u = 0$ (equivalent to the standard KSEC model) and to also encompass the majority of values calculated from the M4 dataset.

For each point in the turbulence parameter hypergrid, five nonstationary stochastic turbulence realizations were generated with different random seeds. The motivation for generating these "redundant realizations" in the previous project was to be able to investigate the variance of a specified wind turbine response statistic as a function of wind parameter inputs. The number of redundant realizations at each hypergrid point was limited by the available HPC hours; five was the largest number of redundant realizations that could be simulated with the desired hypergrid edge vectors. The turbulence spatial grid parameters for the WindPACT 5 MW mode was a 140×140 square grid centered at 154 m with 15 points along each edge. The turbulent fields were simulated with time step of 0.05 s for 630 s so that the first 30 seconds of each simulation could be used to eliminate transient behavior in the wind turbine simulations.

2.2.2. Wind turbine simulation Once the 6,000 stochastic nonstationary turbulent fields were generated, the next step in the previous project was to use them as wind input for the four WindPACT reference models. The WindPACT models were implemented in version 7.02 of FAST (Fatigue, Aerodynamics, Structures, and Turbulence), NREL's open-source wind turbine simulator, with the aerodynamic calculations being performed by v13.00 of AeroDyn. The majority of the FAST structural parameters were taken from the Excel design worksheets from the original WindPACT project that were made available to the author. The control parameters were taken from the "WP 1.5 MW" certification test model that is distributed with the FAST software, which is based on the WindPACT 1.5 MW reference model. The aerodynamic model used a Beddoes-Leishman dynamic stall model, assumed an equilibrium induction model, and featured standard Prandtl loss models at both the hub and tip. The equilibrium induction model was used instead of the generalized dynamic wake model because the latter model was found to be numerically unstable for wind ranges between 6 and 9 m s^{-1}. The turbine response was simulated for 630 s (the duration of the turbulence simulation), and then the first 30 s were discarded to remove transience.

2.2.3. Response statistic calculation From each 10-minute wind turbine simulation, a multitude of reponse statistics were calculated and saved. These response statistics included but were not limited to quantities such as the mean, variance, skewness, kurtosis, maximum, and minimum of many output loads and deflections. Additionally, the damage-equivalent loads (DELs) for many forces/moments were also calculated and stored. The DEL is defined as the load amplitude that would create the same fatigue damage as a particular load history [22]. By extracting these response statistics, the training data thus consisted of thousands of input points (i.e., the turbulence parameters) with corresponding output points (i.e., the response statistics). These data generated in the previous project could then be used for any analysis technique involving predefined input/output data.

3. Wind Turbine Response Surfaces

In this paper, the training data generated in the previous project is used to calibrate a response surface to be used as a surrogate model in Sobol SI calculations. Response surfaces are a well-established technique for calibrating a surrogate model to mimic input/output data with low computational cost (e.g., [12]). In this paper, the input vector $\mathbf{x}$ consists of the four turbulence parameters—U_{ref}, $I_{u,ref}$, $\log_{10}(L_u)$, and ρ_u—and the scalar output y can be any response statistic for any wind turbine. For brevity, this paper will only consider a single response statistic for a single turbine model, viz. the maximum blade root bending moment of the WindPACT 5 MW reference model. However, it is important to note that the proposed analysis methodology can be utilized with any turbine model or response statistic.

3.1. Fitting methodology

There are many sophisticated methods for fitting response surfaces to data (see, e.g., [12]), but the authors have found the following simple technique to produce very good results with the available wind turbine training data:

(i) First, determine the parametric form for the response surface. High-order response surfaces can have any order of individual polynomial terms and cross-terms. The authors found that individual polynomial orders above 5 for U_{ref} and 3 for $I_{u,ref}$, $\log_{10}(L_u)$, and ρ_u resulted in overfitting, whereas individual orders that were smaller did not accurately capture the trends in the data. The cross terms in the parameteric form were determined by taking a full polynomial factorization—which consisted of $6 \times 4 \times 4 \times 4 = 384$ terms—and eliminating cross-terms whose total power exceeded the maximum individual polynomial order of 5. This resulted in a total number of 111 polynomial terms in the parametric form.

(ii) Second, with the parametric form of the response surface determined, the optimal response surface coefficients were determined using ordinary least squares [12]:

$$\boldsymbol{\beta} = \left([X]^T[X]\right)^{-1}[X]\mathbf{y}. \tag{4}$$

Here, $\mathbf{y}$ is the vector of output data and the matrix $[X]$ is constructed from the input data. Each row of $[X]$ corresponds to a given training data point, and each column corresponds to a polynomial term in the parametric form (e.g., $x_1x_2x_3^2x_4$). Once the optimal coefficients $\boldsymbol{\beta}$ are calculated, the response surface is completely determined.

3.2. Evaluation of fit

Before proceeding with the sensitivity analysis, it is important to verify that the calibrated response surface matches the training data. If there are significant errors between the response surface and training data, any conclusions drawn from the calculated load sensitivities will be invalid. Additionally, although the training data were generated in a previous project, it is of interest to investigate whether the selected number of redundant realizations was sufficient or whether more should have been used. This section contains an error analysis for the response surface of interest in this paper: the maximum blade root moment of the WindPACT 5 MW reference model.

The quality of the response surface fit can be examined visually by plotting a subset of the training data and comparing it to the response surface curve. This is demonstrated in the top subplot in Fig. 3 for the WindPACT 5 MW maximum blade root bending moment. The training data and response surface are filtered such that $I_{u,ref} = 0.1$, $\log_{10}(L_u) = 2.5$, and $\rho_u = 0.1$. Note that the multiple points for a single wind speed shown in the figure are the five redundant realizations. The response surface curve passes through the training data points, demonstrating that the surface accurately models the trends in the training data. This is highlighted by the

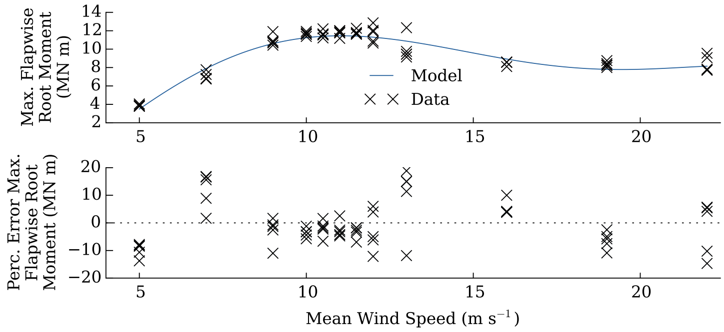

Figure 3. Comparison of response surface with training data for the 10-minute maximum of the flapwise root moment ($I_{u,ref} = 0.1$, $\log_{10}(L_u) = 2.5$, and $\rho_u = 0.1$): (a) model versus data, (b) percent error.

low percent errors between the training data and response surface that are plotted in the bottom subplot. For the entire dataset, 78% of the percent errors are less than 10% and 97% are less than 20%. Because the percent errors are acceptably small, we conclude that the response surface can serve as an accurate surrogate model for in sensitivity calculations.

In addition to an evaluation of the fit, it is also of interest to determine the convergence of the response surface as a function of the number of redundant realizations N_{rr}. This can be done by fitting response surfaces to subselected data and evaluating the error between the response surface and the entire set of training data. This error can be quantified via the normalized sum-of-squared error (NSSE):

$$NSSE = \frac{1}{N_{uniq}} \sum_{i=1}^{N_{uniq}} \left[\hat{y}_{N_{rr}}(\mathbf{x}_i) - \overline{y}(\mathbf{x}_i)\right]^2 . \tag{5}$$

Here, N_{uniq} is the number of unique points in the wind parameter hypergrid (1200 for this paper), $\mathbf{x}_i$ is the ith turbulence parameter coordinate in the hypergrid, $\hat{y}_{N_{rr}}$ is the response surface calibrated to the subselected data, and $\overline{y}$ is the "true" model, calculated by averaging the five redundant realizations in the training data. The N_{rr} training data points subselected at each unique $\mathbf{x}_i$ are chosen randomly for each NSSE calculation.

The results for the calculated NSSE as a function of N_{rr} are shown in Fig. 4. The NSSE was calculated ten times for $N_{rr} = 1$, 2, 3, and 4; there is only one point for $N_{rr} = 5$ because all the data are used to fit the response surface. As expected, the values for the NSSE show the largest magnitudes and variations for $N_{rr} = 1$, and the magnitudes and variation both decrease with more redundant realizations. At $N_{rr} = 4$, the ten NSSE calculations lie almost on top of one another and a take a value that is almost identical to the NSSE for $N_{rr} = 5$. Because the NSSE variation is so small at $N_{rr} = 4$ and the magnitude is extremely similar to the value at $N_{rr} = 5$, we can reasonably conclude that simulating more redundant points would not result in a significant decrease in error between the response surface and the training data. Thus, the response surface can be said to have converged with respect to the number of redundant realizations.

3.3. Sampling from a response surface

Having calibrated the response surface and verified its convergence, the last step in this section is to describe how to use the response surface to sample synthetic load response statistics. One drawback of the response surface is that it is a deterministic model, not a stochastic model.

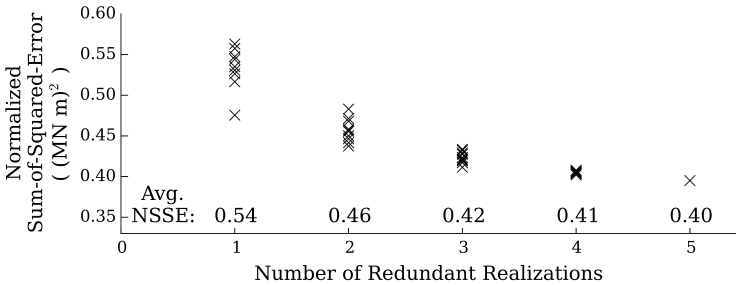

Figure 4. Convergence of error for the maximum flapwise root bending moment for the WindPACT 5 MW model with respect to the number of redundant realizations at each hypergrid point

In other words, for a given set of turbulent parameters $\mathbf{x}$, the response surface returns an approximation of the *average* of the response statistic at that hypergrid coordinate. In many applications, this deterministic characterization is not realistic. To account for this issue, the authors add in Gaussian noise with a 10% coefficient of variation (COV) to the mean loads that are modeled with the response surface. This value of the COV was determined by plotting the empirical COV values from the training data and choosing an upper bound that encompassed the majority of the data. With this added noise, the method to sample stochastic response load statistics from the response surface can be expressed as

$$Y(\mathbf{x}) = \hat{y}(\mathbf{x})(1 + 0.1N), \tag{6}$$

where Y is the stochastic load response statistic, $\hat{y}$ is the response surface evaluated at the selected turbulence parameters, and N is a normally distributed random variable.

4. Sobol Sensitivity Indices

The calibrated response surface can be used to easily calculate Sobol SIs with little computational effort. Sobol SIs are based on the decomposition of variance of a model represented by a square integrable function $f(\mathbf{x})$ defined on the unit hypercube H^n. This function can be expanded using the analysis-of-variance decomposition [23]

$$f(\mathbf{x}) = f_0 + \sum_{s=1}^{n} \sum_{i_1 < \cdots < i_s}^{s} f_{i_1 \cdots i_s}(x_i, \ldots, x_{i_s}). \tag{7}$$

As noted in [10], this decomposition is both unique and orthogonal if

$$\int_{H^n} f_{i_1 \ldots i_s}(x_{i_1}, \ldots, x_{i_s}) dx_{i_k} = 0, \quad 1 \le k \le s. \tag{8}$$

The variances of the decomposition terms sum to the total model variance according to

$$\sigma^2 = \sum_{s=1}^{n} \sum_{i_1 < \cdots < i_s}^{s} \sigma^2_{i_1 \ldots i_s}, \tag{9}$$

where $\sigma^2_{i_1 \ldots i_s} = \int_{H^n} f^2_{i_1 \ldots i_s}(x_{i_1}, \ldots, x_{i_s}) dx_{i_1} \ldots dx_{i_s}$. The global Sobol SIs are then defined as

$$S_{i_1 \ldots i_s} = \frac{\sigma^2_{i_1 \ldots i_s}}{\sigma^2}, \tag{10}$$

and they sum to unity [10].

It is impractical to calculate the global Sobol SIs for each term in the decomposition, so instead the Sobol SIs for subsets of input variables are utilized. Assume that the input variables have been partitioned into two subsets $\mathbf{x} = (\mathbf{y}, \mathbf{z})$. Let K denote the set of input variable indices for which $x_i \in \mathbf{y}$ if $i \in K$. The variance associated with subset $\mathbf{y}$ is then defined as [10]

$$\sigma_y^2 = \sum_{s=1}^{m} \sum_{(i_1<\cdots<i_s)\in K} \sigma_{i_1 \ldots i_s}^2, \tag{11}$$

where m is the number of input variables in subset $\mathbf{y}$. This quantity accounts for all of the variance associated with the decomposition terms that only depend on input variables in $\mathbf{y}$. The variance σ_z^2 is defined similarly for the input variables in $\mathbf{z}$. A complementary concept to these variance definitions is the total variance:

$$(\sigma_y^{tot})^2 = \sigma^2 - \sigma_z^2. \tag{12}$$

The total variance for subset $\mathbf{y}$ is the variance associated with all decomposition terms that have at least one subterm in $\mathbf{y}$. The SIs are analogously defined [10]:

$$S_y = \frac{\sigma_y^2}{\sigma^2} \tag{13}$$

$$S_y^{tot} = \frac{(\sigma_y^{tot})^2}{\sigma^2} = 1 - \frac{\sigma_z^2}{\sigma^2} = 1 - S_z. \tag{14}$$

The global SI for subset $\mathbf{y}$ can be calculated using Monte Carlo integration [10]:

$$S_y \approx \frac{\frac{1}{N}\sum_{i=1}^{N} f(\mathbf{y},\mathbf{z}) f(\mathbf{y},\mathbf{z}') - \left[\frac{1}{N}\sum_{i=1}^{N} f(\mathbf{y},\mathbf{z})\right]^2}{\frac{1}{N}\sum_{i=1}^{N} f^2(\mathbf{y},\mathbf{z}) - \left[\frac{1}{N}\sum_{i=1}^{N} f(\mathbf{y},\mathbf{z})\right]^2}, \tag{15}$$

where $\mathbf{x}' = (\mathbf{y}', \mathbf{z}')$ is a second set of samples of the input parameters that is independent from $\mathbf{x}$ and N is the number of Monte Carlo samples.

The Sobol SIs can be used to eliminate unimportant inputs in a model by considering the effective dimension of f in the truncation sense, d_T. This effective dimension is the smallest integer d_T such that some threshold percentage of the total variance p is maintained [10]:

$$\sum_{0<y\subseteq\{1,2,\ldots,d_T\}} S_y \geq p. \tag{16}$$

If the complementary SI S_z^{tot} is much less than unity, the set of variables $\mathbf{z}$ can be deemed not important and set to nominal values $\mathbf{z}_0$ (see Sec. 5 in [10]). This concept can be applied to wind turbine response statistics to determine which, if any, turbulence parameters have little to no effect on a wind turbine load response statistic.

For this paper, the four turbulence parameters of interest are subgrouped such that $\mathbf{y} = \{U_{ref}, I_{u,ref}\}$ and $\mathbf{z} = \{\log_{10}(L_u), \rho_u\}$. This grouping was chosen based on preliminary first-order Sobol SI calculations for the maximum blade root bending moment of the WindPACT 5 MW reference model, which indicated that the Kaimal length scale and concentration parameter had lower SIs than the mean wind speed and turbulence intensity. The values for S_y and S_z were then calculated using the following procedure:

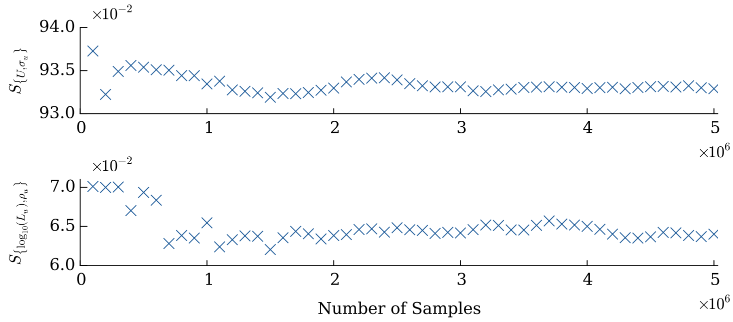

Figure 5. Convergence of $S_{\{U,\sigma_u\}}$ and $S_{\{\log_{10}(L_u),\rho_u\}}$

(i) Draw two sets of $N \times 4$ uniform, independent samples $\mathbf{u}$ and $\mathbf{u}'$ (one to generate $\mathbf{x}$ and one to generate $\mathbf{x}'$).

(ii) Generate $\mathbf{x}$ and $\mathbf{x}'$ by mapping $\mathbf{u}$ and $\mathbf{u}'$ to a correlated probability space with realistic distribution functions using the empirical inverse distributions and correlations provided in [18] for a measurement height of 76 m.

(iii) Use $\mathbf{x}$ and $\mathbf{x}'$ with the response surface calibrated in Sec. 3 to generate $f(\mathbf{y},\mathbf{z})$, $f(\mathbf{y}',\mathbf{z})$, and $f(\mathbf{y},\mathbf{z}')$.

(iv) Calculate S_y according to Eq. (15) and S_z with the complement (i.e., replace $f(\mathbf{y},\mathbf{z}')$ with $f(\mathbf{y}',\mathbf{z})$).

This procedure was repeated for a number of Monte Carlo samples ranging from 100,000 to 5,000,000, and the resulting SIs are plotted in Fig. 5. The variation of the SIs is generally quite small, within 0.01 even for the smallest sample size, and the values converge within 2 million samples to $S_y \approx 0.933$ and $S_z \approx 0.064$. Note that millions of samples can be calculated very easily using response surfaces with sample times on the order of minutes for a basic laptop. From these values for S_y and S_z we can use Eq. (14) to calculate the total SIs for the maximum blade root bending moment for the WindPACT 5 MW reference model:

$$S_y^{tot} \approx 0.936 \quad \text{and} \quad S_z^{tot} \approx 0.067. \tag{17}$$

Because $S_z^{tot} \ll 1$, we can conclude that, for this wind turbine model and response statistic, the Kaimal length scale and concentration parameter do not significantly impact the variation of the wind turbine load and can therefore be set to nominal values. This is an extremely important finding, because it is the first systematic evidence to support modeling wind turbine load response statistics purely by mean wind speed and turbulence standard deviation. It should be cautioned, however, that this does not imply that U_{ref} and $I_{u,ref}$ are the only two turbulence parameters that need to be considered when modeling wind turbine load variation. The findings here simply state that, of the variance caused by the set of turbulence parameters $\{U_{ref}, I_{u,ref}, \log_{10}(L_u), \rho_u\}$, the majority can be attributed to $\{U_{ref}, I_{u,ref}\}$. It is possible that the wind turbine response statistic is highly affected by a turbulence parameter or effect not considered in this study. Additionally, extreme caution should be used when extrapolating these findings to other wind turbine models. Changing the wind turbine model in any way, either structurally or aerodynamically, could result in changed load sensitivities. However, the advantage of the methodology presented in this paper is that it is entirely general and may therefore be applied to any wind turbine model/response statistic of interest.

5. Conclusions

This paper presents a methodology to calibrate response surfaces to wind turbine response statistics data. This response surface can then be used for model-evaluation-heavy calculations such as the calculation of Sobol SIs. Response surfaces are characterized by a high-dimensional polynomial surface that can be calibrated to any set of input/output data and then used to generate synthetic data at a low computational cost. The proposed methodology is demonstrated using training data of the maximum blade root bending moment of the WindPACT 5 MW reference model that was generated for a previous project. The previous project simulated thousands of nonstationary turbulent fields with varying U_{ref}, $I_{u,ref}$, $\log_{10}(L_u)$, and ρ_u and used them as inflow to simulate the responses of the four WindPACT reference models. In this paper, a response surface was calibrated to the maximum blade root bending moment of the WindPACT 5 MW reference model, and the quality of the fit and convergence of the response surface was determined to be satisfactory. The response surface was then used to calculate total Sobol SIs for $\mathbf{y} = \{U_{ref}, I_{u,ref}\}$ and $\mathbf{z} = \{\log_{10}(L_u), \rho_u\}$. The sensitivity indices were found to converge within two million Monte Carlo samples to $S_y^{tot} \approx 0.936$ and $S_z^{tot} \approx 0.067$. The small value of S_z^{tot} means that the output variance due to variation in $\mathbf{z} = \{\log_{10}(L_u), \rho_u\}$ is negligible, and so these two input parameters may be set to nominal values. Thus, the findings in this paper represent the first systematic evidence that supports modeling stochastic wind turbine load response statistics purely by mean wind wind speed and turbulence intensity.

[1] Sathe A, Mann J, Barlas T, Bierbooms W and Bussel G 2013 *Wind Energy* **16** 1013–1032
[2] Sutherland H J 2002 *Journal of solar energy engineering* **124** 432–445
[3] Nelson L D, Manuel L, Sutherland H J and Veers P S 2003 *Journal of solar energy engineering* **125** 541–550
[4] Kelley N 2011 Turbulence-turbine interaction: The basis for the development of the Turbsim stochastic simulator Tech. Rep. NREL/TP-5000-52353 National Renewable Energy Laboratory Golden, CO
[5] Downey R P 2006 *Uncertainty in wind turbine life equivalent load due to variation of site conditions* Master's thesis Technical University of Denmark
[6] Saranyasoontorn K and Manuel L 2008 *Journal of Wind Engineering and Industrial Aerodynamics* **96** 503–523
[7] Dimitrov N, Natarajan A and Kelly M 2015 *Wind Energy* **18** 1917–1931
[8] Mücke T, Kleinhans D and Peinke J 2011 *Wind Energy* **14** 301–316
[9] Sobol I M 2001 *Mathematics and computers in simulation* **55** 271–280
[10] Kucherenko S, Feil B, Shah N and Mauntz W 2011 *Reliability Engineering & System Safety* **96** 440–449
[11] Saltelli A 2002 *Computer Physics Communications* **145** 280–297
[12] Gavin H P and Yau S C 2008 *Structural Safety* **30** 162–179
[13] Iooss B, Van Dorpe F and Devictor N 2006 *Reliability Engineering & System Safety* **91** 1241–1251
[14] Volkova E, Iooss B and Van Dorpe F 2008 *Stochastic Environmental Research and Risk Assessment* **22** 17–31
[15] Malcolm D and Hansen A 2006 WindPACT turbine rotor design study Tech. Rep. NREL/SR-500-32495 National Renewable Energy Laboratory
[16] Griffin D A 2001 Windpact turbine design scaling studies technical area 1 – composite blades for 80-to 120-meter rotor Tech. Rep. NREL/SR-500-29492 National Renewable Energy Laboratory Golden, Colorado
[17] Tangler J L and Somers D M 1995 Nrel airfoil families for hawts *Proceedings of WindPower* (Washington, D.C.: American Wind Energy Association)
[18] Rinker J M, Gavin H P, Clifton A, Veers P S and Kilcher L F 2016 *Boundary-Layer Meteorology*
[19] International Electrotechnical Commission 2005 Wind turbines - part 1: design requirements 3rd ed.
[20] Clifton A 2013 135-m meteorological masts at the National Wind Technology Center: Instrumentation, data acquisition, and processing URL `http://wind.nrel.gov/MetData/Publications/NWTC_135m_MetTowers.pdf`
[21] Rinker J M 2016 *An Empirically Based Stochastic Turbulence Simulator with Temporal Coherence for Wind Energy Applications* Ph.D. thesis Duke University Durham, North Carolina, USA
[22] Veldkamp H F 2006 *Chances in wind energy: a probalistic approach to wind turbine fatigue design* Ph.D. thesis Technische Universiteit Delft
[23] Hoeffding W 1948 *The annals of mathematical statistics* 293–325